The *b*-Coloring of graphs in Bistar and Its Various Properties

Arun kumar A
Assistant professor
Department of Mathematics,
Government first grade college,
Bannur, Karnataka 571101
Email: akumara78@yahoo.co.in

Article History

Article Submission
29 August 2018

Revised Submission
15 October 2018

Article Accepted
29 November 2018

Article Published
31 December 2018

Abstract

A graph's b-coloring iswhenG its vertices are properly colored so that every color class has a vertex connected to at least one vertex in every other color class. In graph Gthe b – chromatic number denoted by $\boldsymbol{\varphi}(G)$*,is the maximal integer k such that G may have a b – coloring with k- colors. In the present study, we derive the b-chromatic number for the corona product of the Bistar graph, which is represented by the following:*$\boldsymbol{\varphi}(B_{n,n} \circ P_n)$, $\boldsymbol{\varphi}(B_{n,n} \circ C_n)$, $\boldsymbol{\varphi}(P_n \circ B_{n,n})$, $\boldsymbol{\varphi}(C_n \circ B_{n,n})$*respectively.*

Keywords: : *b* – chromatic number, *b*-coloring,Bistar graph,path graph,cycle graph..

1. INTRODUCTION

Consider the conventional definitions of appropriate coloring and chromatic number for a given simple graph, G. We can alter the color of these vertices to produce a valid coloring with fewer colors if, in the case that we have a correct coloring of G and there exists a color c such that every vertex v with color c is not next to at least one other color (which may depend on v). This approach can be used repeatedly, but since the coloring issue is NP-hard, we cannot expect to attain the chromatic number of G. Irving and Manlove proposed the concept of b-coloring [1] based on this premise. By definition, a b-coloring is a suitable coloring that is unaffected by the aforementioned heuristic; the b-chromatic number quantifies the worst possible example of this type of coloring. More formally, consider a proper coloring ψ of G. A vertex u is said to be a b-vertex in ψ if u has a neighbor in each color class different from its own. A b-coloring of G is a proper coloring of G such that each color class contains a b-vertex. A basis of a b-coloring ψ is a subset of b-vertices of ψ containing one b-vertex of each color class. The biggest integer b(G) for which G has a b-coloring with b(G) colors is the b-chromatic number of G. It is NP-hard to compute the b-chromatic number of a graph G [1], regardless of whether the graph is chordal [7] or bipartite [6].

As a function of other graph characteristics (clique number, chromatic number, andbiclique number), Kouider and Zaker [2] provided certain upper bounds for the b-chromatic number of several types of graphs. In [3], Kouider

ISSN: 2250-0839

and El Sahili demonstrated that if G is a d-regular graph without cycles of length 6 and with girth 5.then ϕ(G)=d+1..Paths are fundamental concepts of graph theory, described in the introductory sections of most graph theory texts[4] .The corona graph was introduced by Frucht and Harary [5] in1970.In this paper we give some corona product of graphs using proper coloring

2.Preliminaries

Bistar Bn,n is the graph obtained by linked the center (apex) vertices of two copies of K1,n by an edge. The vertex set of Bn,n is V (Bn,n) = {u, v, ui , vi/1 ≤ i ≤ n}, where u, v are apex vertices and ui, vi are pendent vertices. The edge set of Bn,n is E(Bn,n) = {uv, uui , vvi/1 ≤ i ≤ n}. So, |V (Bn,n)| = 2n+2 and |E(Bn,n)| = 2n+1.

A graph with a single cycle, or at least three vertices linked in a closed chain, is referred to as a cycle graph or circular graph. Cn stands for the cycle graph with n vertices. The total number of edges in Cn is equal to the total number of vertices, and each vertex has degree 2, or precisely two edges that intersect it.

A path graph or linear graph is a graph whose vertices can be listed in the order v1, v2, …, vn such that the edges are {vi, vi+1} where i = 1, 2, …, n − 1. Evenly, in a path has at least two vertices is connected and has two terminal vertices (vertices that have degree 1), while all others (if any) have degree 2.

Thecorona productofGandHis the graphG◦Hobtained by taking onecopy ofG, called the center graph,|V(G)|copies ofH, called the outer graph, andmaking theithvertex ofGadjacent to every vertex of theithcopy ofH,where1≤i≤|V(G)|..

3.b-coloring the Bistar graph's Corona graph using the Path graph

Theorem 1:For any positive integer n,n≥ 1 the b-chromatic number of the corona product of Bistar graph with path graph is φ(Bn,n°Pn)=n+2 for n≥ 1.

Proof:

Let V(Bn,n)={u,v,ui,vi:1≤i≤ n}

V(pn)={pi:1≤ i ≤ n}

Let V(Bn,n°Pn)={u}∪{v}∪{ui: 1≤i≤ n}∪{ v_i: 1 ≤ i ≤ n, } ∪{uij: 1≤i≤ n, 1 ≤ j ≤ n}∪
{ vij:1≤i≤ n, 1≤j≤ n}∪{xi: 1≤i≤ n}∪{yi: 1≤i≤ n}

Consider n≥ 1,assign the following n+2 colors asb-chromatic number for Bn,n°Pn

For u ,assign the color c1

For v,assign the color c2

For ui: 1≤i≤ n assign the color cn+2

Forvi:1≤i≤ n assign the color cn+2

For uij assign the l;1≤ l≤ n + 2 colors except the colors of vertices u &{ui:1≤i≤ n}

For vij assign the k;1≤ k≤ n + 2 colors except the colors of vertices v&{vi:1≤i≤ n}

For xi: 1≤i≤ n assign the color ci+1

For yi : 1≤i≤ n assign the color ci+2

This proves that this coloring is a b-Coloring.

The coloring procedure depends the number of vertices of Bistar graph(Bn,n) that is Bn,n has degree n+1.Hence we can assign maximum of n+2 colors to get a b-Coloring.Hence the proof.

4. b-coloring of the Bistar graph's Corona graph using the Cycle graph

Theorem 2:The b-chromatic number of the Corona product of the Bistar graph with the cycle graph, for each positive integer n, where n≥1, isφ (Bn,n°Cn)=n+2 for n≥ 1.

Proof:

Let V(Bn,n)={u,v,ui,vi:1≤i≤ n}

V(Cn)={ci:1≤ i ≤ n}

Let V(Bn,n°Pn)={u}∪{v}∪ {ui: 1≤i≤ n}∪ { v_i: 1 ≤ i ≤ n, } ∪ {uij: 1≤i≤ n, 1 ≤ j ≤ n}
∪ { vij:1≤i≤ n, 1≤j≤ n}

ISSN: 2250-0839

Consider n≥ 1,assign the following n+2 colors asb-chromatic number for Bn,n°Cn.
Assign colorc2 to v.
Assign colorc1 to u.
For ui: 1≤i≤ n assign the color cn+2
Forvi:1≤i≤ n assign the color cn+2
For uij assign the color ck: 1 ≤ k ≤ n + 2 except the colors of vertices u &{ui:1≤i≤ n}
For vij assign the color cm;1≤m≤ n + 2 colors except the colors of vertices v&{vi:1≤i≤ n}

This coloring is a b-Coloring, as the outcome demonstrates.
The coloring process is dependent on the Bistar graph's (Bn,n)vertex count. Degree n+1 is assigned to the bistar graph (Bn,n). Therefore, to obtain a b-Coloring, we can assign a maximum of n+2 colors. Thus, the evidence.

5. Path graph's Corona graph's b-coloring using the Bistar graph
Theorem 3:The b-chromatic number of the Corona product of the Path graph with the Bistar graph, for each positive integer n, where n≥1, is φ(Pn°Bn,n)=n+2 for n= 1.
= n+3 for n≥2
Proof:
Let V(pn)={pi:1≤ i ≤ n}
V(Bn,n)={u,v,ui,vi:1≤i≤ n}
Let V(Pn° Bn,n)={pi: 1≤i≤ n}∪{ui: 1≤i≤ n}∪ { v_i: 1 ≤ i ≤ n, }
∪ {uij: 1≤i≤ n, 1 ≤ j ≤ n}∪ { vij:1≤i≤ n, 1≤j≤ n}
Consider the following cases
Case 1:
Assign the following n+2 colors to P1° B1,1as the b-chromatic number for n=1.

For p1 ,assign the color c1
For u1 ,assign the color c2
For v1 ,assign the color c3
For u11assign the color c3
For v11assign the color c2
The above shows that this coloring is a b-Coloring.
Case2:
For any positive integer n,n≥ 2
Assign the following n+3 colors as b-chromatic for Pn°Bn,n,n≥2
For pi: 1≤i≤ n assign the color ci
For ui: 1≤i≤ n assign the color cn+1
For vi:1≤i≤ n − 1 assign the color cn+2
For vn assign the colors cn+3
For uij and vij : 1≤i≤ n , 1≤j≤ n assign the colors cd;1≤d≤ n + 3 as by the proper coloring way.
The above shows that this coloring is a b-Coloring.
The coloring procedure depends the number of vertices of Bistar graph(Bn,n).
Here the Bn,n has degree n+2.Hence we can assign maximum of n+3 colors to get a b-Coloring.
Hence the proof.

6.b-coloring the Bistar graph with the Corona graph of the Path graph

ISSN: 2250-0839

Theorem 4:The b-chromatic number of the Corona product of the Cycle graph with Bistar graph, for each positive integer n, where n≥1, is φ(Cn° Bn,n)=n+3 for n≥ 1.

Proof:

Let V(Cn)={di:1≤ i ≤ n}

V(Bn,n)={u,v,ui,vi:1≤i≤ n}

Let

V(Cn°Bn,n) ={di: 1≤i≤ n}∪{ui: 1≤i≤ n}∪ { Vi: 1 ≤ i ≤ n, }

∪ {uij: 1≤i≤ n, 1 ≤ j ≤ n}∪ { vij:1≤i≤ n, 1≤j≤ n}

Assuming n≥1, designate the subsequent n+3 hues as the b-chromatic number for Cn° Bn,n

For di: 1≤i≤ n assign the color ci

For ui: 1≤i≤ n assign the color cn+1

Forvi:1≤i≤ n-1assign the color cn+2

And for vnassign the color cn+3

For uij assign the colors ci:1≤i≤n+3 except the colors of vertices ui& di:1≤i≤ n

For vij assign ci:1≤i≤n+3 the colors except the colors of vertices vi& di:1≤i≤ n

It indicates that b-coloring is used in this coloration. The coloring process is dependent on the Bistar graph's Bn,nvertex count. The degree of the Bn,n is n+2.Therefore, to obtain a b-Coloring, we can assign a maximum of n+3 colors. Thus, the evidence.

.

References

1. Harary, F. (1956), "Structural balance: A generalization of Heider's theory", PsychologicalReview, 63 (5): 277 293.
2. Zaslavsky, Thomas (1998), "A mathematical bibliography of signed and gain graphs and allied areas", Electronic Journal of Combinatorics, 5, Dynamic Surveys 8, 124 pp.
3. L. Zhong, "‚ harmonic index for graphs," Applied Mathematics Letters, vol. 25, no. 3, pp. 561
4. O Ravi, R Senthil Kumar, A Hamari Choudhi, Weakly ⊐ g-closed sets, Bulletin Of The International Mathematical Virtual Institute, 4, Vol. 4(2014), 1-9
5. O Ravi, R Senthil Kumar, Mildly Ig-closed sets, Journal of New Results in Science, Vol3,Issue 5 (2014) page 37-47
6. O Ravi, A senthilkumar R & Hamari Choudhi, Decompositions of Ï g-Continuity via Idealization, Journal of New Results in Science, Vol 7, Issue 3 (2014), Page 72-80.
7. O Ravi, A Pandi, R Senthil Kumar, A Muthulakshmi, Some decompositions of πg-continuity, International Journal of Mathematics and its Application, Vol 3 Issue 1 (2015) Page 149-154.
8. S. Tharmar and R. Senthil Kumar, Soft Locally Closed Sets in Soft Ideal Topological Spaces, Vol 10, issue XXIV(2016) Page No (1593-1600).

ISSN: 2250-0839