A Study on Single Valued Neutrosophic Graphs: Dominance

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Abstract

To create the single valued neutrosophic graph G=(A,B), we take a crisp graph G=(V,E) and replace its vertex and edge sets with single valued neutrosophic sets. The single valued neutrosophic graph G=(A,B) has a dominating set and dominant number, which we define in this work. We also note certain constraints on the dominating number of G=(A,B)

Keywords: neutrosophic set, neutrosophic graph , dominating set, domination number.

1. INTRODUCTION

In actuality, the majority of issues are related to inconsistent, ambiguous, and missing information. One effective mathematical technique for handling these kinds of issues is the neutrosopic set. Smarandache introduces neutrosophic sets [6,7]. Fuzzy sets, intuitionistic fuzzy sets, and interval valued intuitionistic fuzzy sets are all generalized into neutrosophic sets. A generalization of intuitionistic fuzzy sets, the single valued neutrosophic sets have three independent membership functions whose values fall within [0,1]. Applied mathematics heavily relies on graph theory, and one of the main areas of graph theory research is dominance. To create the single valued neutrosophic graph G=(A,B), we take a crisp graph G=(V,E) and replace its vertex and edge sets with single valued neutrosophic sets

In this paper, we define dominating set and dominating number of the single valued neutrosophic graph *G=(A,B)* also observe some bounds of the dominating number of *G=(A,B).*

2. PRELIMINARIES

Definition 2.1. Let X be a space of points (objects) with generic elements in X denoted by x, then the neutrosophic set A (NS A) is an object having the form

$$A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$$

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where the functions $T, I, F \rightarrow \left]^{-}0,1^{+}\right[$ define respectively the a truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element $x \in X$ to the set A with the condition

$$^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$$

The functions $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $\left]^{-}0,1^{+}\right[$.

Definition 2.2. A single valued neutrosophic graphs *G=(A,B)* with vertex set V is defined by the set vertex and edge membership functions. The vertex membership functions

$$T_A : V \rightarrow [0,1]$$
$$I_A : V \rightarrow [0,1]$$
$$F_A : V \rightarrow [0,1]$$

T_A , denote the truth membership function , I_A, denote the indeterminacy membership function and F_A, denote the falsity membership function of the vertices $v_i \in V$ and $0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$ for all $v_i \in V$

The edge membership functions

$$T_B : E \subseteq V \times V \rightarrow [0,1]$$
$$I_B : E \subseteq V \times V \rightarrow [0,1]$$
$$F_B : E \subseteq V \times V \rightarrow [0,1]$$

are defined by

$$T_B(v_i v_j) \leq \wedge \left\lfloor T_A(v_i), T_A(v_j) \right\rfloor$$
$$I_B(v_i v_j) \geq \vee \left[I_A(v_i), I_A(v_j) \right]$$
$$F_B(v_i v_j) \geq \vee \left[F_A(v_i), F_A(v_j) \right]$$

T_B , denote the truth membership function, I_B, denote the indeterminacy membership function and F_B, denote the falsity membership function of the edges $(v_i v_j) \in E$ and $0 \leq T_B(v_i v_j) + I_B(v_i v_j) + F_B(v_i v_j) \leq 3$ for all $(v_i v_j) \in E$.

Definition 2.3. In a single value neutrosophic graph $G = (A, B)$, any two vertices v_i *and* v_j are said to be adjacent, if

$$T_B(v_i v_j) = \wedge \left\lfloor T_A(v_i), T_A(v_j) \right\rfloor$$
$$I_B(v_i v_j) = \vee \left[I_A(v_i), I_A(v_j) \right]$$
$$F_B(v_i v_j) = \vee \left[F_A(v_i), F_A(v_j) \right]$$

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The set of all adjacent vertices of a vertex v_i are neighbourhood set of the vertex v_i, and it is denoted by $N(v_i)$.

Definition 2.4. In a single value neutrosophic graph $G=(A,B)$, a vertices v_i is said to be isolated vertex, if there no edges adjacent on v_i.

Definition 2.5. In a single value neutrosophic graph $G=(A,B)$, a vertices v_i is said to be Pendent vertex, if there only one edges adjacent on v_i. A vertex in a single valued neutrosophic graph adjacent to the pendent vertex is called a support of the pendent edge.

Definition 2.6.A single value neutrosophic graph $G=(A,B)$ is said to be complete there is a strong edge between every pair of vertices.

$$T_B(v_iv_j) = \wedge\left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_iv_j) = \vee\left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_iv_j) = \vee\left[F_A(v_i), F_A(v_j)\right]$$

For every $v_i, v_j \in V$.

Definition 2.7. The cardinality of a vertex $v_i \in V$ in a single value neutrosophic graph $G=(A,B)$ is defined by $|v_i| = T_A(v_i)+I_A(v_i)+F_A(v_i)$

Definition 2.8. The neighbourhood T-degree of a vertex $v_i \in V$ in a single value neutrosophic graph $G=(A,B)$ is defined by $d_T(v_i)=\sum_{i\neq j}|v_j|$, for every vertex $v_j \in V$ such that v_iv_j is a T-adjacent vertices. (i.e)

$$T_B(v_iv_j) = \wedge\left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_iv_j) \geq \vee\left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_iv_j) \geq \vee\left[F_A(v_i), F_A(v_j)\right]$$

Definition 2.9. The neighbourhood I-degree of a vertex $v_i \in V$ in a single value neutrosophic graph $G=(A,B)$ is defined by $d_I(v_i)=\sum_{i\neq j}|v_j|$, for every vertex $v_j \in V$ such that v_iv_j is a I-adjacent vertices. (i.e)

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$$T_B(v_iv_j) \leq \wedge \left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_iv_j) = \vee \left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_iv_j) \geq \vee \left[F_A(v_i), F_A(v_j)\right]$$

Definition 2.10. The neighbourhood F-degree of a vertex $v_i \in V$ in a single value neutrosophic graph $G = (A, B)$ is defined by $d_F(v_i) = \sum_{i \neq j} \left|v_j\right|$, for every vertex $v_j \in V$ such that v_iv_j is a F-adjacent vertices. (i.e)

$$T_B(v_iv_j) \leq \wedge \left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_iv_j) \geq \vee \left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_iv_j) = \vee \left[F_A(v_i), F_A(v_j)\right]$$

Definition 2.11. The neighbourhood degree of a vertex $v_i \in V$ in a single value neutrosophic graph $G = (A, B)$ is defined by $d_N(v_i) = \sum_{i \neq j} \left|v_j\right|$, for every vertex $v_j \in V$ such that

$$T_B(v_iv_j) = \wedge \left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_iv_j) = \vee \left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_iv_j) = \vee \left[F_A(v_i), F_A(v_j)\right]$$

Definition 2.12. The neighbourhood degree of a vertex $v_i \in V$ in a single value neutrosophic graph $G = (A, B)$ is defined by $td(v_i) = d_N(v_i) + \left|v_i\right|$

Definition 2.13. In a single value neutrosophic graph $G = (A, B)$ .If $v_i \in V$ is said to be a strong neighbour of vertex $v_j \in V$ such that

a) v_i *and* v_j are adjacent vertices in $G = (A, B)$
b) $d_N(v_i) \geq d_N(v_j)$

Definition 2.14. The order of the single value neutrosophic graph $G = (A, B)$ is defined by $O(G) = \sum_{i \in V} \left|v_i\right|$

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Definition 2.15. The cardinality of an edge $v_i v_j \in E$ in a single value neutrosophic graph $G = (A, B)$ is defined by $|v_i v_j| = T_B(v_i v_j) + I_B(v_i v_j) + F_B(v_i v_j)$

Definition 2.16. The size of the single value neutrosophic graph $G = (A, B)$ is defined by

$$S(G) = \sum_{i,j \in V} |v_i v_j|$$

3. DOMINATION IN SNGLE VALUED NEUTROSOPHIC GRAPH

In this section we define dominating set and dominating number of SVNG and also observe some bounds and properties of domination number

Definition 3.1. In a single value neutrosophic graph $G = (A, B)$, if any two vertices v_i and v_j are said to be T-dominating each other such that there is a T-strong edge between the vertices v_i and v_j.

$$T_B(v_i v_j) = \wedge \left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_i v_j) \geq \vee \left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_i v_j) \geq \vee \left[F_A(v_i), F_A(v_j)\right]$$

Definition 3.2. In a single value neutrosophic graph $G = (A, B)$, the set $D \subseteq V$ is said to be a T-dominating set of $G = (A, B)$ such that every vertex $v \in V - D$ is T-dominated by at least one vertex $u \in D$.

Definition 3.3. In a single value neutrosophic graph $G = (A, B)$, the T-dominating set $D \subseteq V$ is said to be a minimal T-dominating set of $G = (A, B)$ such that $D - \{v\}$ is not a T-dominating set in $G = (A, B)$ for all $v \in D$.

Definition 3.4. In a single value neutrosophic graph $G = (A, B)$, the T-domination number is defined by the minimum cardinality among all the minimal T-dominating set of $G = (A, B)$. It is denoted by $\gamma_T(G)$.

Definition 3.5. In a single value neutrosophic graph $G = (A, B)$, if any two vertices v_i and v_j are said to be I-dominating each other such that there is a I-strong edge between the vertices v_i and v_j.

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$$T_B(v_iv_j) \leq \wedge \left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_iv_j) = \vee \left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_iv_j) \geq \vee \left[F_A(v_i), F_A(v_j)\right]$$

Definition 3.6. In a single value neutrosophic graph $G = (A, B)$, the set $D \subseteq V$ is said to be a I-dominating set of $G = (A, B)$ such that every vertex $v \in V - D$ is I-dominated by at least one vertex $u \in D$.

Definition 3.7. In a single value neutrosophic graph $G = (A, B)$, the I-dominating set $D \subseteq V$ is said to be a minimal I-dominating set of $G = (A, B)$ such that $D - \{v\}$ is not a I-dominating set in $G = (A, B)$ for all $v \in D$.

Definition 3.8. In a single value neutrosophic graph $G = (A, B)$, the I-domination number is defined by the minimum cardinality among all the minimal I-dominating set of $G = (A, B)$. It is denoted by $\gamma_I(G)$.

Definition 3.9. In a single value neutrosophic graph $G = (A, B)$, if any two vertices v_i and v_j are said to be F-dominating each other such that there is a F-strong edge between the vertices v_i and v_j.

$$T_B(v_iv_j) \leq \wedge \left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_iv_j) \geq \vee \left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_iv_j) = \vee \left[F_A(v_i), F_A(v_j)\right]$$

Definition 3.10. In a single value neutrosophic graph $G = (A, B)$, the set $D \subseteq V$ is said to be a F-dominating set of $G = (A, B)$ such that every vertex $v \in V - D$ is F-dominated by at least one vertex $u \in D$.

Definition 3.11. In a single value neutrosophic graph $G = (A, B)$, the F-dominating set $D \subseteq V$ is said to be a minimal F-dominating set of $G = (A, B)$ such that $D - \{v\}$ is not a F-dominating set in $G = (A, B)$ for all $v \in D$.

Definition 3.12. In a single value neutrosophic graph $G = (A, B)$, the F-domination number is defined by the minimum cardinality among all the minimal F-dominating set of $G = (A, B)$. It is denoted by $\gamma_F(G)$.

ISSN: 2250-0839

Definition 3.13. In a single value neutrosophic graph $G=(A,B)$, if any two vertices v_i and v_j are said to be dominating each other such that there is a strong edge between the vertices v_i and v_j.

Definition 3.14. In a single value neutrosophic graph $G=(A,B)$, the set $D \subseteq V$ is said to be a dominating set of $G=(A,B)$ such that every vertex $v \in V-D$ is dominated by at least one vertex $u \in D$.

Definition 3.15. In a single value neutrosophic graph $G=(A,B)$, the dominating set $D \subseteq V$ is said to be a minimal dominating set of $G=(A,B)$ such that $D-\{v\}$ is not a dominating set in $G=(A,B)$ for all $v \in D$.

Definition 3.16. In a single value neutrosophic graph $G=(A,B)$, the domination number is defined by the minimum cardinality among all the minimal dominating set of $G=(A,B)$. It is denoted by $\gamma_{SVN}(G)$.

Example 3.1.

Consider the single valued neturosophic graph G=(A,B) . Let A be a single valued neturoscophic subset of V and let B single valued neturoscophic subset of E defined b

	A	b	c	d	e	f
T_A	.2	.2	.1	.4	.4	.3
I_A	.7	.2	.2	.6	.7	.4
F_A	.4	.7	.4	.2	.3	.3

	ab	ac	bc	bd	ce	de	ef
T_B	.2	.1	.1	.2	.1	.3	.3
I_B	.7	.7	.3	.6	.7	.8	.7
F_B	.7	.4	.7	.7	.5	.4	.3

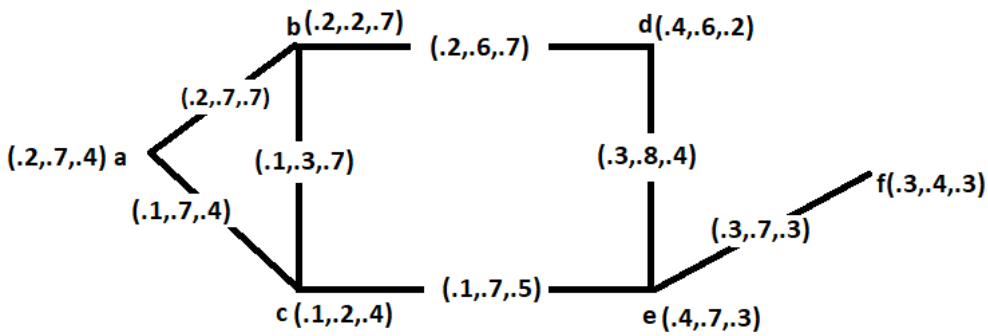

Figure 3.1, *G(A,B)*

In Figure 3.1,

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- (*a* and *b*), (*b* and *d*) and (*e* and *f*) are dominated vertices
- (*a* and *b*), (*b* and *c*) , (*b* and *d*), (*c* and *e*) and (*e* and *f*) are T-dominated vertices
- (*a* and *b*), (*a* and *c*) , (*b* and *d*), (*c* and *e*) and (*e* and *f*) are I-dominated vertices
- (*a* and *b*), (*a* and *c*) , (*b* and *c*), (*b* and *d*) and (*e* and *f*) are F-dominated vertices
- The set $\{b, f\}$, $\{b, c, f\}$, $\{b, f\}$ and $\{a, c, f\}$ is a T-dominating set, I-dominating set, F-dominating set and dominating set of G=(A,B) respectively.
- In G=(A,B) , $\gamma_T(G) = 2.1$, $\gamma_I(G) = 2.8$, $\gamma_F(G) = 2.1$ and $\gamma_{SVN}(G) = 3.0$

Theorem 3.1.

A single valued neutrosophic graph *G=(A,B)* with minimum dominating set $D \subseteq V$. Then the set $V - D$ is the dominating set of *G=(A,B).*

Proof

Let $D - \{v\}$ be the minimum dominating set of *G = (A,B),*and if D is minimum dominating set then $D - \{v\}$ is not an dominating set of *G = (A,B)*. Clearly every vertex $v_i \in V - D$ is adjacent to the vertex in $D - \{v\}$

$$\begin{aligned} T_B(v_i v_j) &= \wedge \left[T_A(v_i), T_A(v_j)\right] \\ I_B(v_i v_j) &= \vee \left[I_A(v_i), I_A(v_j)\right] \qquad atleast\ a\, vertex v_j \cup D \\ F_B(v_i v_j) &= \vee \left[F_A(v_i), F_A(v_j)\right] \end{aligned}$$

Clearly every vertex $v_j \in D$ is adjacent to vertex in $v_i \in V - D$,therefore *V-D* is a dominating set of *G = (A,B).*

Hence proved.

Theorem 3.2.

A single valued neutrosophic graph *G=(A,B)*then $\gamma_{SVN}(G) \le 3\left[\frac{O(G)}{2}\right]$.

Proof

Let $D \subseteq V$ be the minimum dominating set of *G = (A,B)* and by theorem 1 $V - D$ is a dominating set vertex having $\Delta_N(G) = d(v_i)$

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$$\gamma_{SVN}(G) = Min\left(|D|, |V-D|\right)$$
$$\gamma_{SVN}(G) \le \frac{O(G)}{2} \qquad \because V = D \cup (V-D)$$
$$\gamma_{SVN}(G) \le \frac{3O(G^*)}{2} \qquad \because O(G) \le 3\left[O(G^*)\right]$$

Hence proved.

Theorem 3.3.

Let D be a minimal dominating set of a single valued neutrosophic graph *G=(A,B)* if and only if every vertex $v \in D$ satisfies at least one of the following property.

1) There exists an edge $v \in V - D$ such that $N(v) \cap D = \{u\}$.
2) "u" is not dominated by the vertex in D.

Proof:

First we assume that D satisfies (i) or (ii) .

In (i), we note that u is the only vertex dominates v. In (ii), u is not dominated by the edges in D. Therefore we get $D - \{u\}$ is not a dominating set of G. This implies D is a minimal dominating set of G. Conversely, we assume that D is a minimal dominating set of G. Clearly $D - \{u\}$ is not a dominating set of G. This implies there exist a vertex v $\in$ V - $(D - \{u\})$ which is not dominated by the vertex in $D - \{u\}$.

If $u = v$, we get u is adjacent to no vertex in D. If u is adjacent to vertices in D, obviously u is dominated by vertex in D. This implies $D - \{u\}$ is a dominating set of G. This contradict to our assumption D is minimal. Therefore e_1 is dominated by no vertices in D, property (ii) holds. If $u \ne v$, v is dominated by the vertices in D. Since D is a dominating set of G. The vertex v is not dominated by the vertices in $D - \{e_1\}$. Therefore we get $N(v) \cap D = \{u\}$. Hence proved

Theorem 3.4.

A single valued neutrosophic graph *G=(A,B)* with minimum dominating set $D \subseteq V$. Let $v_i \in V$ is the vertex having $\Delta_N(G) = d(v_i)$ then $\gamma_{SVN}(G) \le O(G) - d(v_i)$

Proof

Let $D \subseteq V$ be the minimum dominating set of *G = (A,B)* and $v_i \in V$ be the vertex having $\Delta_N(G) = d(v_i)$. If $v_i \notin D$, there is a vertex the minimum $v_j \in D$ such that ,

$$T_B(v_i v_j) = \wedge \left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_i v_j) = \vee \left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_i v_j) = \vee \left[F_A(v_i), F_A(v_j)\right]$$

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Therefore $V - N(v_i)$ is a dominating set of *G=(A,B)*

$$\gamma_{SVN}(G) \leq |V - N(v_i)|$$
$$\gamma_{SVN}(G) \leq O(G) - d(v_i)$$

Hence proved.

Conclusion

Fuzzy sets and intuitionistic fuzzy sets are generalized to become neutrosophic sets. Neutrosophic models are superior than classical, fuzzy, and intuitionistic fuzzy models in terms of precision, flexibility, and compatibility with the system. In the next study, we intend to expand our research to other dominations parameters in single valued neutrosophic graphs and determine the boundaries of the domination parameters. In this work, we have established dominating set, minimum dominating set, and domination number.

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