

An Arithmetic-Geometric Index for Different Signed Graphs

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<i>Article History</i>	<i>Abstract</i>
Article Submission 25 February 2018 Revised Submission 30 April 2018 Article Accepted 30 May 2018 Article Published 30 June 2018	A topological index, which is often referred to as a connectivity index, is a molecular structure descriptor that describes the topology of a chemical molecule based on its molecular network. Different topological indices are divided into groups according to their degree, spectrum, and distance. In this investigation, we computed and examined degree-based topological indices, including the positive and negative arithmetic-geometric indices (and). Index) and index in regular graphs, complete graphs, complete bipartite graphs, union of graphs, and join of graphs are further examined and derived. Provide examples to further clarify the theory Keywords: Graphs, arithmetic-geometric index, and topological index.

1. INTRODUCTION

A signed graph is defined by an ordered pair $\Sigma = (G, \sigma)$ where $G = (V, E)$ is an underlying graph of Σ and $\sigma : E \rightarrow \{+, -\}$ is a function called a signature function.

The positive degree of the vertex u in the signed graph is defined by number of positive edges are incident in the vertex u and it is denoted by $d_+(u)$. The negative degree of the vertex u in the signed graph is defined by number of negative edges are incident in the vertex u and it is denoted by $d_-(u)$.

The maximum positive degree of the signed graph Σ is maximum positive degree along the vertices in Σ it is denoted by $\Delta_+(G)$. The maximum negative degree of the signed graph Σ is maximum negative degree along the vertices in Σ it is denoted by $\Delta_-(G)$.

Note that the sum of positive degree and negative degree of a vertex in $u \in \Sigma$ is the degree of vertex in underlying graph $G = (V, E)$.

The positive degree of the edge uv in the signed graph is defined by number of positive edges are adjacent to the edge uv and it is denoted by $d_+(uv)$. The negative degree of

the edge uv in the signed graph is defined by number of negative edges are adjacent to the edge uv and it is denoted by $d_-(uv)$.

The minimum positive degree of the signed graph Σ is minimum positive degree along the edges in Σ it is denoted by $\delta_{E+}(G)$. The minimum negative degree of the signed graph Σ is minimum negative degree along the edges in Σ it is denoted by $\delta_{E-}(G)$.

Defined in terms of the degrees of the vertices of a graph, degree-based topological indices are one of the most researched classes of topological indices used in mathematical chemistry. The definition of such a topological index may be expressed as follows:

$$TI(G) = \sum_{pq \in G} F(d(p), d(q))$$

$$\text{Arithmetic-geometric index is defined as } AG(G) = \sum_{pq \in E(G)} \left(\frac{(d_p + d_q)}{2\sqrt{(d_p \cdot d_q)}} \right).$$

We define the Arithmetic-geometric index of signed graphs in this study. We also look at various limits and characteristics of the signed graph's arithmetic-geometric index.

2. ARITHMETIC-GEOMETRIC INDEX IN SIGNED GRAPHS

The positive and negative AG index in signed graphs are defined in this section, along with the index's characteristics and boundaries examined.

Definition 2.1: The positive Arithmetic-geometric (AG^+) index of signed graphs is defined as $AG^+(\Sigma) = \sum_{pq \in E(G)} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right)$.

Definition 2.2: The negative Arithmetic-geometric (AG^-) index of signed graphs is defined as $AG^-(\Sigma) = \sum_{pq \in E(G)} \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right)$.

Theorem 2.1: In a positive K regular signed graph with n vertices, then the (AG^+) index is $AG^+(\Sigma) \geq \left(\frac{nK}{2} \right)$

Proof: Let Σ be a positive K regular signed graph with n vertices. Therefore we get $d_-(v_i) = K$, for every $v_i \in \Sigma$. The positive Arithmetic-geometric (AG^+) index of signed graphs

$$\text{is defined as } AG^+(\Sigma) = \sum_{pq \in E(G)} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right).$$

$$\begin{aligned}
 AG^+(\Sigma) &= \sum_{pq \in E(G)} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\
 &= \sum_{pq \in E(G)} \left(\frac{(K + K)}{2\sqrt{(K \cdot K)}} \right) \\
 &= \sum_{pq \in E(G)} \left(\frac{(2K)}{2\sqrt{(K^2)}} \right) \\
 AG^+(\Sigma) &= \sum_{pq \in E(G)} (1).
 \end{aligned}$$

In a negative K regular signed graph Σ with n vertices, there is minimum $\left(\frac{nK}{2}\right)$ edges in Σ .

$$AG^+(\Sigma) \geq 1+1+1+\dots\left(\frac{nK}{2}\right) \text{ Times}$$

$$AG^+(\Sigma) \geq \left(\frac{nK}{2}\right).$$

$$\text{Hence } AG^+(\Sigma) \geq \left(\frac{nK}{2}\right)$$

Illustration 2.1: Positive 2-regular signed graph $\Sigma(G, \sigma)$.

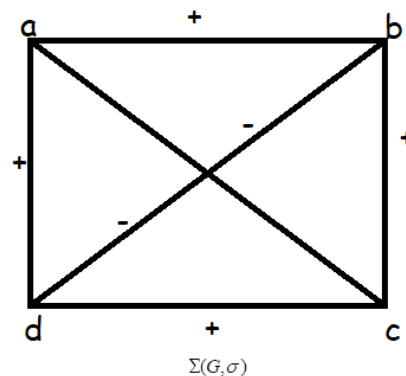


Figure 2.1: Positive 2-regular signed graph $\Sigma(G, \sigma)$.

In a positive 2-regular signed graph $\Sigma(G, \sigma)$ the positive degree of the every vertices in $\Sigma(G, \sigma)$ is 2.. The order $\Sigma(G, \sigma)$ is $O(\Sigma) = n = 4, S(G) = 6$. The (AG^+) index

$$AG^+(\Sigma) = 4 \geq \left(\frac{nK}{2}\right).$$

$$\begin{aligned}
 AG^+(\Sigma) &= \sum_{pq \in E(G)} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\
 AG^+(\Sigma) &\geq \left(\frac{(d_a^+ + d_b^+)}{2\sqrt{(d_a^+ \cdot d_b^+)}} \right) + \left(\frac{(d_b^+ + d_c^+)}{2\sqrt{(d_b^+ \cdot d_c^+)}} \right) + \left(\frac{(d_c^+ + d_d^+)}{2\sqrt{(d_c^+ \cdot d_d^+)}} \right) + \left(\frac{(d_a^+ + d_d^+)}{2\sqrt{(d_a^+ \cdot d_d^+)}} \right) \\
 AG^+(\Sigma) &\geq \left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right) + \left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right) + \left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right) + \left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right) \\
 &= 4 \left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right) = 4 \left(\frac{4}{2\sqrt{4}} \right)
 \end{aligned}$$

$$AG^+(\Sigma) \geq 4$$

Theorem 2.2: In a negative K regular signed graph with n vertices, then the (AG^-) index is $AG^-(\Sigma) \geq \left(\frac{nK}{2} \right)$

Proof: Let Σ be a negative K regular signed graph with n vertices. Therefore we get $d_-(v_i) = K$, for every $v_i \in \Sigma$. The negative Arithmetic-geometric (AG^-) index of signed graphs is defined as

$$\begin{aligned}
 AG^-(\Sigma) &= \sum_{pq \in E(G)} \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) \\
 AG^-(\Sigma) &= \sum_{pq \in E(G)} \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) \\
 &= \sum_{pq \in E(G)} \left(\frac{(K + K)}{2\sqrt{(K \cdot K)}} \right) \\
 &= \sum_{pq \in E(G)} \left(\frac{(2K)}{2\sqrt{(K^2)}} \right) \\
 AG^-(\Sigma) &= \sum_{pq \in E(G)} (1).
 \end{aligned}$$

In a negative K regular signed graph Σ with n vertices, there is minimum $\left(\frac{nK}{2} \right)$ edges in Σ .

$$AG^-(\Sigma) \geq 1+1+1+\dots\left(\frac{nK}{2}\right) \text{Times}$$

$$AG^-(\Sigma) \geq \left(\frac{nK}{2}\right).$$

$$\text{Hence } AG^-(\Sigma) \geq \left(\frac{nK}{2}\right)$$

Illustration 2.2: Negative 2-regular signed graph $\Sigma(G, \sigma)$.

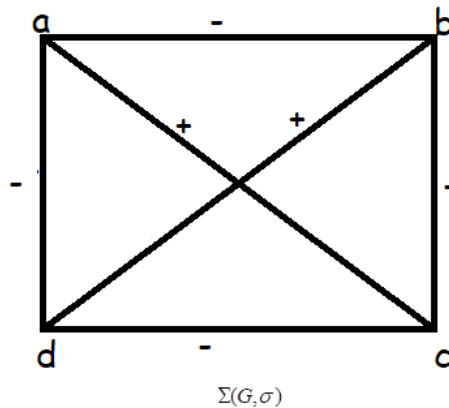


Figure 2.2: Negative 2-regular signed graph $\Sigma(G, \sigma)$.

In a negative 2-regular signed graph $\Sigma(G, \sigma)$ the negative degree of the every vertices in $\Sigma(G, \sigma)$ is 2.. The order $\Sigma(G, \sigma)$ is $O(\Sigma) = n = 4, S(G) = 6$. The (AG^-) index

$$AG^-(\Sigma) = 4 \geq \left(\frac{nK}{2}\right).$$

$$AG^-(\Sigma) = \sum_{p,q \in E(G)} \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right).$$

$$AG^+(\Sigma) \geq \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) + \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) + \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) + \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right)$$

$$\begin{aligned} AG^-(\Sigma) &\geq \left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right) + \left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right) + \left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right) + \left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right) \\ &= 4 \left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right) = 4 \left(\frac{4}{2\sqrt{4}} \right) \end{aligned}$$

$$AG^-(\Sigma) \geq 4$$

Theorem 2.3: For a positive complete signed graph of n vertices, then the (AG) index

$$AG^+(\Sigma) \geq \frac{n(n-1)}{2}.$$

Proof: Let $\Sigma(G, \sigma)$ be a positive complete signed graph this implies the induced sub graph $\langle V^+ \rangle$ is a complete graph of order $O(G) = n$. This implies the positive degree of every vertices in G is $(n-1)$ and n number of vertices in the induced subgraph $\langle V^+ \rangle$. In a $(n-1)$ regular graph there is $\frac{n(n-1)}{2}$ edges in a complete graph of n vertices. The positive Arithmetic-geometric (AG^+) index of signed graphs is defined as

$$\begin{aligned}
 AG^+(\Sigma) &= \sum_{pq \in E(G)} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\
 AG^+(\Sigma) &= \sum_{\substack{pq \in E^+ \\ p, q \in V^+}} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) + \sum_{\substack{pq \notin E^+ \\ p, q \notin V^+}} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\
 AG^+(\Sigma) &\geq \sum_{\substack{pq \in E^+ \\ p, q \in V^+}} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\
 AG^+(\Sigma) &\geq \sum_{\substack{pq \in E^+ \\ p, q \in V^+}} \left(\frac{((n-1) + (n-1))}{2\sqrt{((n-1)^2)}} \right) \\
 AG^+(\Sigma) &\geq \sum_{\substack{pq \in E^+ \\ p, q \in V^+}} \left(\frac{(2(n-1))}{2\sqrt{((n-1)^2)}} \right) = \sum_{\substack{pq \in E^+ \\ p, q \in V^+}} \left(\frac{(2(n-1))}{2(n-1)} \right) \\
 AG^+(\Sigma) &\geq \sum_{\substack{pq \in E^+ \\ p, q \in V^+}} (1) = 1 + 1 + 1 + \dots \frac{n(n-1)}{2} \text{ times} \\
 AG^+(\Sigma) &\geq \frac{n(n-1)}{2} \\
 \text{Hence } AG^+(\Sigma) &\geq \frac{n(n-1)}{2}.
 \end{aligned}$$

Illustration 2.3: Positive complete signed graph $\Sigma(G, \sigma)$.

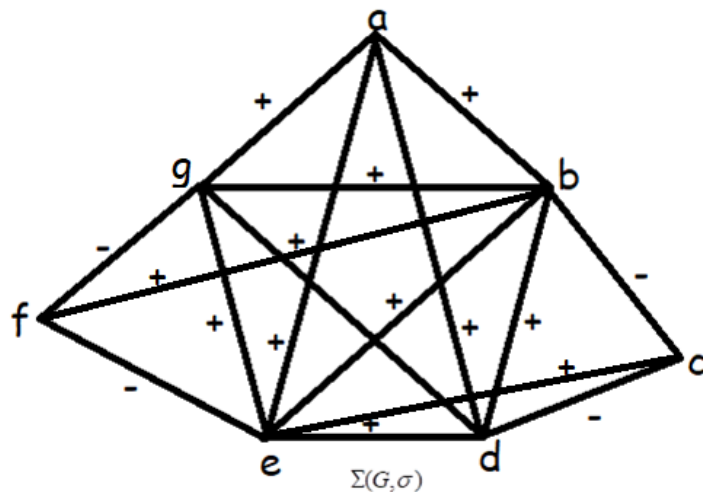


Figure 2.3: Positive complete signed graph $\Sigma(G, \sigma)$.

In a positive complete signed graph $\Sigma(G, \sigma)$ the positive degree of the every vertices in $\langle V^+ \rangle$ is 4.. The order $\Sigma(G, \sigma)$ is $O(\Sigma) = n = 7, S(\Sigma) = 14$. The (AG^+) index) $AG^+(\Sigma) = 15 \geq \left(\frac{n(n-1)}{2} \right)$.

$$\begin{aligned}
 AG^+(\Sigma) &= \sum_{pq \in E(G)} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\
 AG^+(\Sigma) &= \sum_{\substack{pq \in E^+(G) \\ p, q \in V^+(G)}} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) + \sum_{\substack{pq \notin E^+(G) \\ p, q \notin V^+(G)}} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\
 AG^+(\Sigma) &= \sum_{\substack{pq \in E^+(G) \\ p, q \in V^+(G)}} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) + \\
 &\quad \left(\frac{(d_b^+ + d_c^+)}{2\sqrt{(d_b^+ \cdot d_c^+)}} \right) + \left(\frac{(d_c^+ + d_d^+)}{2\sqrt{(d_c^+ \cdot d_d^+)}} \right) + \left(\frac{(d_e^+ + d_f^+)}{2\sqrt{(d_e^+ \cdot d_f^+)}} \right) + \left(\frac{(d_f^+ + d_g^+)}{2\sqrt{(d_f^+ \cdot d_g^+)}} \right) \\
 AG^+(\Sigma) &= \sum_{pq \in E^+(G)} \left(\frac{(4+4)}{2\sqrt{(4 \cdot 4)}} \right) + \left(\frac{(4+1)}{2\sqrt{(4)}} \right) + \left(\frac{(4+1)}{2\sqrt{(4)}} \right) + \left(\frac{(4+1)}{2\sqrt{(4)}} \right) + \left(\frac{(4+1)}{2\sqrt{(4)}} \right) \\
 AG^+(\Sigma) &= \sum_{pq \in E^+(G)} \left(1 \right) + \left(\frac{5}{4} \right) + \left(\frac{5}{4} \right) + \left(\frac{5}{4} \right) + \left(\frac{5}{4} \right) \\
 AG^+(\Sigma) &= 1+1+1+.. \left(\frac{5(5-1)}{2} \right) \text{ times} + 4 \left(\frac{5}{4} \right) \\
 AG^+(\Sigma) &= 10+5=15
 \end{aligned}$$

Theorem 2.3: For a negative complete signed graph of n vertices, then the (AG^-) index $AG^-(\Sigma) \geq \frac{n(n-1)}{2}$.

Proof: Let $\Sigma(G, \sigma)$ be a negative complete signed graph this implies the induced sub graph $\langle V^- \rangle$ is a complete graph of order $O(G) = n$. This implies the positive degree of every vertices in G is $(n-1)$ and n number of vertices in the induced sub graph $\langle V^- \rangle$. In a $(n-1)$ regular graph there is $\frac{n(n-1)}{2}$ edges in a complete graph of n vertices. The positive arithmetic-geometric (AG^-) index of signed graphs is defined as

$$\begin{aligned}
 AG^-(\Sigma) &= \sum_{pq \in E(G)} \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) \\
 AG^-(\Sigma) &= \sum_{\substack{pq \in E^- \\ p, q \in V^-}} \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) + \sum_{\substack{pq \notin E^- \\ p, q \notin V^-}} \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) \\
 AG^-(\Sigma) &\geq \sum_{\substack{pq \in E^- \\ p, q \in V^-}} \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) \\
 AG^-(\Sigma) &\geq \sum_{\substack{pq \in E^- \\ p, q \in V^-}} \left(\frac{((n-1) + (n-1))}{2\sqrt{((n-1)^2)}} \right) \\
 AG^-(\Sigma) &\geq \sum_{\substack{pq \in E^- \\ p, q \in V^-}} \left(\frac{(2(n-1))}{2\sqrt{((n-1)^2)}} \right) = \sum_{\substack{pq \in E^- \\ p, q \in V^-}} \left(\frac{(2(n-1))}{2(n-1)} \right) \\
 AG^-(\Sigma) &\geq \sum_{\substack{pq \in E^- \\ p, q \in V^-}} (1) = 1 + 1 + 1 + \dots \frac{n(n-1)}{2} \text{ times} \\
 AG^-(\Sigma) &\geq \frac{n(n-1)}{2} \\
 \text{Hence } AG^-(\Sigma) &\geq \frac{n(n-1)}{2}.
 \end{aligned}$$

Illustration 2.4: Negative complete signed graph $\Sigma(G, \sigma)$.

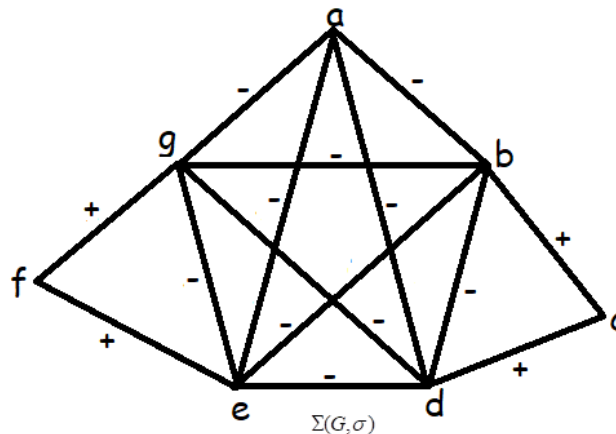


Figure 2.4: Negative complete signed graph $\Sigma(G, \sigma)$.

In a negative complete signed graph $\Sigma(G, \sigma)$ the negative degree of the every vertices in $\langle V^- \rangle$ is 4.. The order $\Sigma(G, \sigma)$ is $O(\Sigma) = n = 7, S(\Sigma) = 14$. The (AG^-) index $AG^-(\Sigma) = 15 \geq \left(\frac{n(n-1)}{2} \right)$.

$$\begin{aligned}
 AG^-(\Sigma) &= \sum_{pq \in E(G)} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\
 AG^-(\Sigma) &= \sum_{pq \in E^+(G)} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) + \sum_{pq \in E^-(G)} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\
 AG^-(\Sigma) &= \left(\frac{(d_b^+ + d_c^+)}{2\sqrt{(d_b^+ \cdot d_c^+)}} \right) + \left(\frac{(d_c^+ + d_d^+)}{2\sqrt{(d_c^+ \cdot d_d^+)}} \right) + \left(\frac{(d_e^+ + d_f^+)}{2\sqrt{(d_e^+ \cdot d_f^+)}} \right) + \left(\frac{(d_f^+ + d_g^+)}{2\sqrt{(d_f^+ \cdot d_g^+)}} \right) + \\
 &\quad \sum_{pq \in E^-(G)} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\
 AG^-(\Sigma) &= \left(\frac{(4+1)}{2\sqrt{(4)}} \right) + \left(\frac{(4+1)}{2\sqrt{(4)}} \right) + \left(\frac{(4+1)}{2\sqrt{(4)}} \right) + \left(\frac{(4+1)}{2\sqrt{(4)}} \right) + \sum_{pq \in E^-(G)} \left(\frac{(4+4)}{2\sqrt{(4 \cdot 4)}} \right) + \\
 AG^-(\Sigma) &= \left(\frac{5}{4} \right) + \left(\frac{5}{4} \right) + \left(\frac{5}{4} \right) + \left(\frac{5}{4} \right) + \sum_{pq \in E^-(G)} (1) \\
 AG^-(\Sigma) &= 4 \left(\frac{5}{4} \right) + \left(1+1+1+.. \left(\frac{5(5-1)}{2} \right) \text{times} \right) \\
 AG^-(\Sigma) &= 5 + 10 = 15
 \end{aligned}$$

Theorem 2.5: For a positive complete bipartite signed graph $K_{m,n}$, then the (AG^+) index $AG^+(\Sigma) \geq \frac{1}{2}(n+m)\sqrt{mn}$.

Proof: Let $\Sigma(G, \sigma)$ be a positive complete bipartite signed graph of vertex sets V_m^+ & V_n^+ respectively. This implies the induced sub graph $\langle V_{m,n}^+ \rangle$ is a positive complete bipartite graph of vertex sets V_m & V_n . This implies the positive degree of every vertices in V_m and V_n are n & m respectively such that $d^+(v_i) = n, \forall v_i \in V_m$ and $d^+(v_j) = m, \forall v_j \in V_n$, there is mn edges in a positive complete bipartite signed graph $K_{m,n}$ of (m, n) vertices. Therefore (AG^+) index

$$\begin{aligned} AG^+(\Sigma) &= \sum_{pq \in E(G)} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\ AG^+(\Sigma) &= \sum_{\substack{pq \in E^+(G) \\ p, q \in V^+(G)}} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) + \sum_{\substack{pq \notin E^+(G) \\ p, q \notin V^+(G)}} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\ AG^+(\Sigma) &= \sum_{\substack{pq \in K_{m,n} \\ p \in V_m \text{ \& } q \in V_n}} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) + \sum_{pq \notin K_{m,n}} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\ AG^+(\Sigma) &\geq \sum_{\substack{pq \in K_{m,n} \\ p \in V_m \text{ \& } q \in V_n}} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\ &= \sum_{\substack{pq \in K_{m,n} \\ p \in V_m \text{ \& } q \in V_n}} \left(\frac{(n+m)}{2\sqrt{(n \cdot m)}} \right) \\ &= \frac{(n+m)}{2\sqrt{(n \cdot m)}} + \frac{(n+m)}{2\sqrt{(n \cdot m)}} + \frac{(n+m)}{2\sqrt{(n \cdot m)}} + \dots mn \text{ times} \\ &= \frac{(n+m)}{2\sqrt{(n \cdot m)}} (mn) \\ AG^+(\Sigma) &\geq \frac{1}{2}(n+m)\sqrt{mn} \end{aligned}$$

$$\text{Hence } AG^+(\Sigma) \geq \frac{1}{2}(n+m)\sqrt{mn}.$$

Illustration 2.5: Positive complete bipartite signed graph $\Sigma(G, \sigma)$.

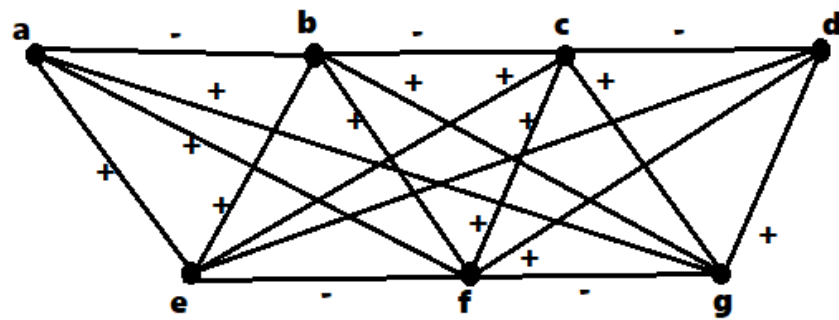


Figure 2.5: Positive complete bipartite signed graph $\Sigma(G, \sigma)$.

In a positive complete bipartite signed graph $\Sigma(G, \sigma)$ the positive degree of the every vertices in V_1^+ and V_2^+ are $m=4$ and $n=3$ respectively. The order $\Sigma(G, \sigma)$ is $O(\Sigma) = 7$. There is 12 edges in positive complete bipartite signed graph $\Sigma(G, \sigma)$. The (AG^+) index

$$AG^+(\Sigma) = 7\sqrt{3} \geq \frac{1}{2}(n+m)\sqrt{mn}.$$

$$\begin{aligned} AG^+(\Sigma) &\geq \sum_{\substack{pq \in K_{m,n} \\ p \in V_m \text{ \& } q \in V_n}} \left(\frac{(d_p^+ + d_q^+)}{2\sqrt{(d_p^+ \cdot d_q^+)}} \right) \\ &= \sum_{\substack{pq \in K_{m,n} \\ p \in V_m \text{ \& } q \in V_n}} \left(\frac{(4+3)}{2\sqrt{(4 \cdot 3)}} \right) \\ &= \frac{(7)}{2\sqrt{(12)}} + \frac{(7)}{2\sqrt{(12)}} + \frac{(7)}{2\sqrt{(12)}} + \dots m \text{ times} \\ &= \frac{(7)}{4\sqrt{3}}(12) \\ AG^+(\Sigma) &\geq 7\sqrt{3} \end{aligned}$$

Theorem 2.6: For a negative complete bipartite signed graph $K_{m,n}$, then the (AG^-) index $AG^-(\Sigma) \geq \frac{1}{2}(n+m)\sqrt{mn}$.

Proof: Let $\Sigma(G, \sigma)$ be a negative complete bipartite signed graph of vertex sets V_m^- & V_n^- respectively. This implies the induced sub graph $\langle V_{m,n}^- \rangle$ is a negative complete bipartite graph of vertex sets V_m & V_n . This implies the negative degree of every vertices in V_m and V_n are n & m respectively such that $d^-(v_i) = n, \forall v_i \in V_m$ and $d^-(v_j) = m, \forall v_j \in V_n$. clearly there is mn edges in a negative complete bipartite signed graph $K_{m,n}$ of (m, n) vertices. Therefore (AG^-) index

$$\begin{aligned}
 AG^-(\Sigma) &= \sum_{pq \in E(G)} \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) \\
 AG^-(\Sigma) &= \sum_{\substack{pq \in E^+(G) \\ p, q \in V^+(G)}} \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) + \sum_{\substack{pq \notin E^+(G) \\ p, q \in V^+(G)}} \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) \\
 AG^+(\Sigma) &= \sum_{\substack{pq \in K_{m,n} \\ p \in V_m \text{ \& } q \in V_n}} \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) + \sum_{pq \notin K_{m,n}} \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) \\
 AG^+(\Sigma) &\geq \sum_{\substack{pq \in K_{m,n} \\ p \in V_m \text{ \& } q \in V_n}} \left(\frac{(d_p^- + d_q^-)}{2\sqrt{(d_p^- \cdot d_q^-)}} \right) \\
 &= \sum_{\substack{pq \in K_{m,n} \\ p \in V_m \text{ \& } q \in V_n}} \left(\frac{(n+m)}{2\sqrt{(n \cdot m)}} \right) \\
 &= \frac{(n+m)}{2\sqrt{(n \cdot m)}} + \frac{(n+m)}{2\sqrt{(n \cdot m)}} + \frac{(n+m)}{2\sqrt{(n \cdot m)}} + \dots m \text{ times} \\
 &= \frac{(n+m)}{2\sqrt{(n \cdot m)}} (mn) \\
 AG^-(\Sigma) &\geq \frac{1}{2} (n+m) \sqrt{mn} \\
 \text{Hence } AG^-(\Sigma) &\geq \frac{1}{2} (n+m) \sqrt{mn} .
 \end{aligned}$$

Illustration 2.6: Negative complete bipartite signed graph $\Sigma(G, \sigma)$.

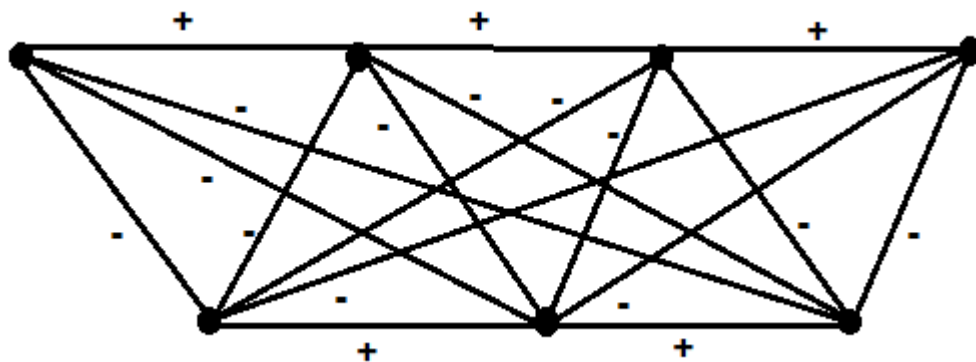


Figure 2.6: Negative complete bipartite signed graph $\Sigma(G, \sigma)$.

In a Negative complete bipartite signed graph $\Sigma(G, \sigma)$ the Negative degree of the every vertices in V_1^+ and V_2^+ are $m=4$ and $n=3$ respectively. The order $\Sigma(G, \sigma)$ is $O(\Sigma) = 7$. There is 12 edges in Negative complete bipartite signed graph $\Sigma(G, \sigma)$. The (AG^+) index $AG^-(\Sigma) = 7\sqrt{3} \geq \frac{1}{2}(n+m)\sqrt{mn}$.

3. Conclusion

In this investigation, we computed and examined degree-based topological indices, including the positive and negative arithmetic-geometric indices (and). Index) and index in regular graphs, complete graphs, complete bipartite graphs, union of graphs, and join of graphs are further examined and derived. Provide examples to further clarify the theory. We shall examine the many degree-based topological indices in the future.

References

1. Harary, F. (1956), "Structural balance: A generalization of Heider's theory", *Psychological Review*, 63 (5): 277-293.
2. Zaslavsky, Thomas (1998), "A mathematical bibliography of signed and gain graphs and allied areas", *Electronic Journal of Combinatorics*, 5, Dynamic Surveys 8, 124 pp.
3. F. Harary, *Graph Theory*, Addison Wesley, Reading, Mass., 1972.
4. Bollobas, *Modern Graph Theory*, Springer Science and Business Media, Berlin, Germany, 2013.
5. Y. Yuan, B. Zhou, and N. Trinajstić, "On geometric-arithmetic ' index," *Journal of Mathematical Chemistry*, vol. 47, no. 2, pp. 833–841, 2010.
6. L. Zhong, " , harmonic index for graphs," *Applied Mathematics Letters*, vol. 25, no. 3, pp. 561
7. O Ravi, R Senthil Kumar, A Hamari Choudhi, [Weakly \$\perp\$ g-closed sets](#), *Bulletin Of The International Mathematical Virtual Institute*, 4, Vol. 4(2014), 1-9
8. O Ravi, R Senthil Kumar, Mildly Ig-closed sets, *Journal of New Results in Science*, Vol3, Issue 5 (2014) page 37-47
9. O Ravi, A senthil kumar R & Hamari Choudhi, Decompositions of $\tilde{I}$ g-Continuity via Idealization, *Journal of New Results in Science*, Vol 7, Issue 3 (2014), Page 72-80.
10. O Ravi, A Pandi, R Senthil Kumar, A Muthulakshmi, Some decompositions of π g-continuity, *International Journal of Mathematics and its Application*, Vol 3 Issue 1 (2015) Page 149-154.
11. S. Tharmar and R. Senthil Kumar, Soft Locally Closed Sets in Soft Ideal Topological Spaces, Vol 10, issue XXIV(2016) Page No (1593-1600).